**CONTINUITY OF FUNCTIONS OF ONE VARIABLE**

The following problems involve the CONTINUITY OF A FUNCTION OF ONE VARIABLE. Function *y* = *f*(*x*) is continuous at point *x*=*a* if the following three conditions are satisfied :

i.) *f*(*a*) is defined ,

ii.) $ \displaystyle{ \lim_{ x \to a } \ f(x) } $exists

and

iii.) $ \displaystyle{ \lim_{ x \to a } \ f(x) } = f(a) $.

Function *f* is said to be continuous on an interval *I* if *f* is continuous at each point *x* in *I*. Here is a list of some well-known facts related to continuity :

1. The SUM of continuous functions is continuous.

2. The DIFFERENCE of continuous functions is continuous.

3. The PRODUCT of continuous functions is continuous.

4. The QUOTIENT of continuous functions is continuous at all points *x* where the DENOMINATOR IS NOT ZERO.

5. The FUNCTIONAL COMPOSITION of continuous functions is continuous at all points *x* where the composition is properly defined.

6. Any polynomial is continuous for all values of *x*.

7. Function *ex* and trigonometry functions $ \sin x $and $ \cos x $are continuous for all values of *x*.

*1 :* Determine if the following function is continuous at *x*=1 .

$ f(x) = \cases{ 3x-5 ,& if $\space x \ne 1 $\space \cr
\ \ \ \ 2 \ \ \ ,& if $ x = 1 $\space } $

*2 :* Determine if the following function is continuous at *x*=-2 .

$ f(x) = \cases{ x^2+2x ,& if $\space x \le -2 $\space \cr
x^3-6x ,& if $ x > -2 $\space } $

*3 :* Determine if the following function is continuous at *x*=0 .

$ f(x) = \cases{ \ \ \ \displaystyle{ x-6 \over x-3 } ,& if $\space x < 0 $\spac...
... 2 \ \ \ \ ,& if $ x = 0 $\space \cr
\sqrt{ 4 + x^2 },& if $ x > 0 $\space } $

*4 :* Determine if the function $ h(x) = \displaystyle{ x^2 + 1 \over x^3+1 } $ is continuous at *x*=-1 .

*5 :* Check the following function for continuity at *x*=3 and *x*=-3 .

$ f(x) = \cases{ \ \ \ \displaystyle{ x^3-27 \over x^2-9 } ,& if $\space x \ne 3...
...r
\ \ \ \ \ \ \ \ \displaystyle{ 9 \over 2 } \ \ \ \ ,& if $ x = 3 $\space } $

*6 :* For what values of *x* is the function $ f(x) = \displaystyle{ x^2 + 3x + 5 \over x^2 + 3x -4 } $continuous ?

*7 :* For what values of *x* is the function $ g(x) = ( \sin(x^{20}+5) )^{1/3} $continuous ?

*8 :* For what values of *x* is the function $ f(x) = \sqrt{ x^2 - 2x } $continuous ?

*10 :* For what values of *x* is the function $ f(x) = \displaystyle{ e^{ \sin x } \over 4 - \sqrt{ x^2 - 9 } } $continuous ?

*11 :* For what values of *x* is the following function continuous ?

$ f(x) = \cases{ \displaystyle{ x-1 \over \sqrt{ x } - 1 } \ ,& if $\space x > 1...
... x \le 1 $\space \cr
\displaystyle{ 6 \over x-4 } \ ,& if $ x < -2 $\space } $

*12 :* Determine all values of the constant *A* so that the following function is continuous for all values of *x* .

$ f(x) = \cases{ A^2 x - A \ ,& if $\space x \ge 3 $\space \cr
\ \ \ \ \ \ 4 \ \ \ ,& if $ x < 3 $\space } $

*13 :* Determine all values of the constants A and B so that the following function is continuous for all values of *x* .

$ f(x) = \cases{ \ \ \ \ Ax - B \ ,& if $\space x \le -1 $\space \cr
2x^2 + 3A...
...f $ -1 < x \le 1 $\space \cr
\ \ \ \ \ \ \ \ 4 \ \ \ ,& if $ x > 1 $\space } $

*14 :* Show that the following function is continuous for all values of *x* .

$ f(x) = \cases{ e^{ -1/x^2 } ,& if $\space x \ne 0 $\space \cr
\ \ \ \ 0 \ \ \ ,& if $ x = 0 $\space } $